2021 PAPER

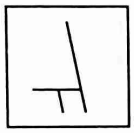
IMO

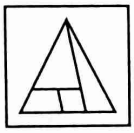
SOF INTERNATIONAL MATHEMATICS OLYMPIAD

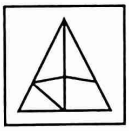
Each question in Achievers Section carries 3 marks, whereas all other questions carry one mark each.

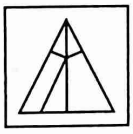
LOGICAL REASONING

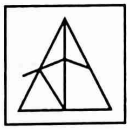
1. In which of the following options, given figure is exactly embedded as one of its parts?



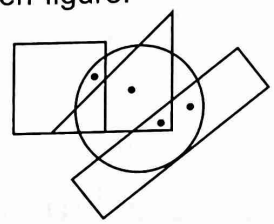
A. 

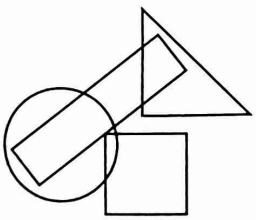
B. 

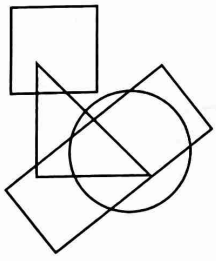
C. 

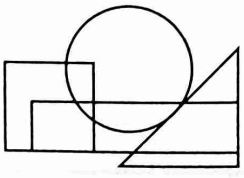
D. 

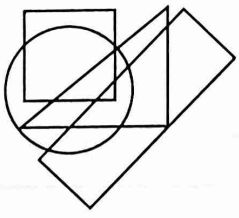
2. Select a figure from the options which satisfies the same conditions of placement of the dots as in the given figure.



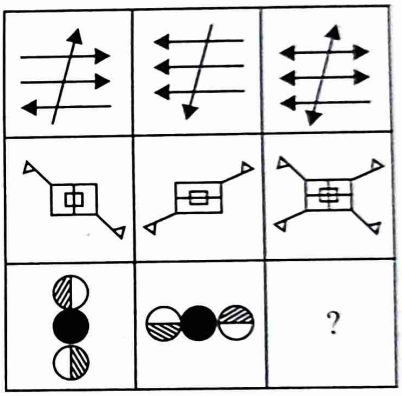
A. 

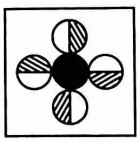
B. 

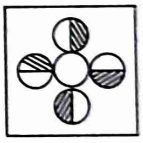
C. 

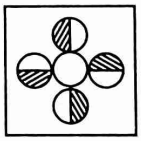
D. 

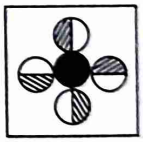
3. Find the figure from the options which will complete the given figure matrix.



A. 

B. 

C. 

D. 

4. In the given series of letters and numbers, the terms follow certain definite pattern in groups. However, some terms in the series are missing, which are given in that order as one of the options below it. Choose the correct option.

2F4, 3 __ 6, __L __, 5 __10, 6 R __

A. G78N12 B. G59M14

C. I57O14 D. I48O12

5. An electronic device rearranges numbers and words in a particular rule in each step. The following is an illustration of input and steps of rearrangement.

Input : window 27 goods 43 door 17 open 32 shut 50

Step I : 17 window 27 goods 43 door open 32 shut 50

Step II : 17 window 27 shut goods 43 door open 32 50

Step III : 17 window 27 shut 32 goods 43 door open 50

Step IV : 17 window 27 shut 32 open goods 43 door 50

Step V : 17 window 27 shut 32 open 43 goods door 50

Step VI : 17 window 27 shut 32 open 43 goods 50 door

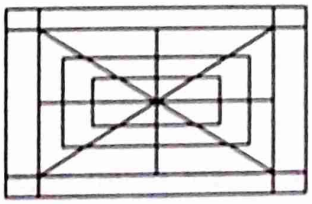
Step VI is the last step of the given input.

As per the rule followed in the above steps, which of the following will be the fourth step for the given input?

Input : cat 62 pull 81 the 22 34 bell

A. 22 the 34 pull cat 62 81 bell
B. the 34 cat 62 pull 81 bell
C. 22 the cat 62 pull 81 34 bell
D. 22 cat 62 pull 81 34 bell

6. Find the number of rectangles formed in the given figure.

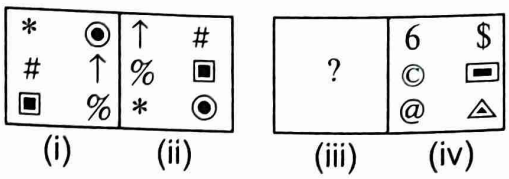


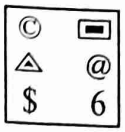
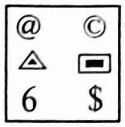
A. 32 B. 33
C. 35 D. More than 35

7. If '@' stands for '×', '#' stands for '+', '*' stands for '+' and '%' stands for '–', then which of the following options becomes incorrect?

A. 20 * 6 @ 9 # 3 % 8 = 30
B. 18 # 6 @ 4 * 15 % 9 = 18
C. 12 * 3 @ 27 # 3 % 16 = 23
D. 19 * 5 % 8 @ 3 = 10

8. There is a certain relationship between figures (i) and (ii). Establish a similar relationship between figures (iii) and (iv) by selecting a suitable figure from the options which will replace the (?) in figure (iii).



A. 
B. 
C. 
D. 

9. Six families M, N, O, P, Q and R are living on different floors of a building. The ground floor is considered as 1st floor and top floor is 6th floor.

Study the following information carefully and answer the given question.

(i) Family R is living above the floor of family M.
(ii) There are 4 families living between P and Q where Q is living on the 1st floor.
(iii) N is living just below the floor of family P.
(iv) Family O is living on the even number floor but not just above family Q.

Which family is living between the family N and family R?

A. M B. Q C. O D. P

10. If 'A $ B' means A is the brother of B; 'A @ B' means A is the wife of B; 'A # B' means A is the daughter of B and 'A * B' means A is the father of B, then which of the following indicates that U is the mother-in-law of P?

A. P@Q$T#U@W B. P@W$Q*T#U
C. P@Q$W*T#U D. P@Q$T#W*U

11. If it is possible to make a meaningful English word with the second, fifth, sixth and eighth letters of the word GENEROUS, then how many words can be formed by using these letters?

A. 0 B. 2
C. 1 D. More than 2

12. Choose the correct mirror image of given figure, if mirror is placed vertically to the left.

A. 

B. 

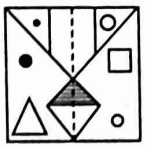
C. 

D. 

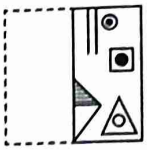
13. If 'North-West' is called 'East', 'South-West' is called 'North' and 'North-East' is called 'South', then what will 'West' called?

A. North-East B. North-West
C. South-East D. South-West

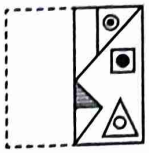
14. A transparent Sheet (X) with a pattern and a dotted line on it is given. Select a figure from the options as to how the pattern would appear when the sheet is folded along the dotted line.

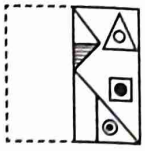


Sheet (X)

A. 

B. 

C. 

D. 

15. Some letters and numbers are coded as given below.

Number or letter	9	M	7	B	R	5	K	4	C
Code	#	@	+	÷	ζ	$	©	%	*

While coding the given number or letter following conditions are also be observed.

Conditions:

I. If any letter is followed and preceded by a number then it is coded as →.

II. If any letter is followed and preceded by a letter then it is codes as ↑.

Which of the following will be the code of 5KR7CM?

A. $©ζ↑@* B. $©ζ→*@
C. $©ζ+*@ D. $©ζ↑*@

MATHEMATICAL REASONING

16. The roots of the quadratic equation $13x^2 + 5x - 2 = 0$ are

A. Real and equal B. Rational
C. Real and distinct D. Not real

17. From a well shuffled pack of 52 cards, a card is drawn at random. Find the probability of getting a red queen.

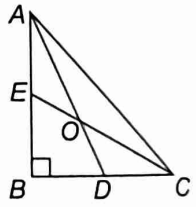
A. $\frac{1}{13}$ B. $\frac{1}{26}$
C. $\frac{4}{13}$ D. $\frac{1}{4}$

18. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the major arc.

A. 30° B. 45°
C. 60° D. 65°

19. In the given figure, AD and CE are the angle bisectors of $\angle A$ and $\angle C$ respectively. If $\angle ABC = 90°$, then find $\angle AOC$.

A. 135°
B. 45°
C. 82°
D. 145°



20. If two dice are rolled together, then find the probability that the sum of the numbers on the two dice is less than or equal to 10.

A. $\frac{11}{12}$ B. $\frac{1}{12}$ C. $\frac{7}{12}$ D. $\frac{1}{9}$

21. The mid point P of line segment joining the points $A(-10, 4)$ and $B(-2, 0)$ lies on the line segment joining the points $C(-9, -4)$ and $D(-4, 6)$. Find the ratio in which P divides CD.

A. 1 : 4 B. 2 : 3 C. 4 : 1 D. 3 : 2

22. Simplify : $\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}} + \frac{2\sqrt{3}}{\sqrt{6}+2}$

A. $\sqrt{2}$ B. $\sqrt{3}$ C. 1 D. 0

23. A right circular cylinder whose diameter is equal to its height, is inscribed in a right circular cone of base diameter 16 cm and height 3 times the base diameter. The axes of both solids coincide. What is the volume (in cm^3) of the solid inside the cone but outside the cylinder?

A. 296π B. 512π

C. 432π D. 592π

24. The length of the sides of a triangle are 4 cm, 6 cm and 8 cm. The length of perpendicular from the vertex opposite to the side whose length is 8 cm, is equal to

A. $\frac{3}{4}\sqrt{15}$ cm B. $\frac{\sqrt{3}}{5}$ cm

C. $\frac{3\sqrt{5}}{4}$ cm D. $\frac{5\sqrt{3}}{4}$ cm

25. If 5^{th} and 17^{th} terms of an A.P. are 46 and 130 respectively, then find its 14^{th} term.

A. 116 B. 109 C. 123 D. 95

26. If the median of the distribution given below is 35. Then find the value of x.

C.I.	10-20	20-30	30-40	40-50	50-60	60-70	70-80
f_i	7	x	6	4	8	3	2

A. 8 B. 10 C. 12 D. 7

27. $\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A}$ is equal to

A. $\frac{2}{\sin^2 A - \cos^2 A}$ B. $\frac{1}{\sin^2 A - \cos^2 A}$

C. 0 D. $\frac{4}{\sin^2 A - \cos^2 A}$

28. Which of the following statements is incorrect?

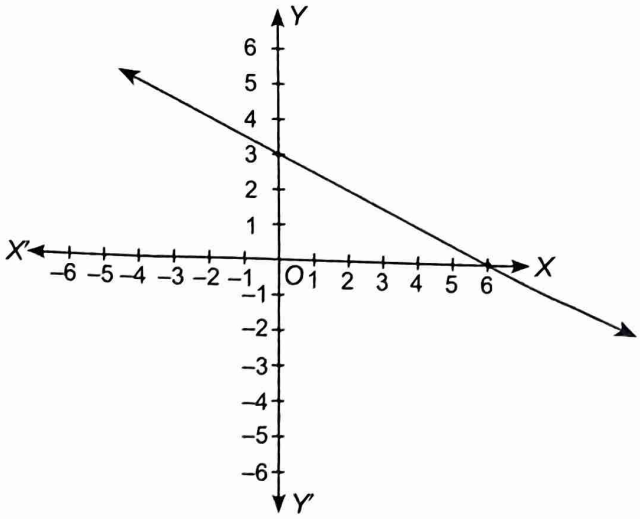
A. The rational form of $17.\bar{6}$ is $\frac{53}{3}$.

B. 0.423442344423... is a rational number.

C. The fractional form of $16 + 2.\bar{9}$ is $\frac{19}{1}$.

D. $\sqrt{25} + \sqrt{64}$ is a rational number.

29. Which of the following options represents the equation of line in the graph?



A. $x + 2y - 6 = 0$ B. $2x + y - 5 = 0$

C. $x - 2y + 7 = 0$ D. $3x - y + 6 = 0$

30. For what value of 'p' the system of equations
$px + 3y = p - 3$
$12x + y = p$
will have no solution?

A. 36 B. 15 C. 12 D. 40

31. Two similar triangles ΔRED and ΔSKY have their area in ratio of 16 : 49. If perimeter of ΔSKY is 21 cm and measures of two sides of ΔRED are 3 cm and 5 cm, then measure of its third side is

A. 4 cm B. 13 cm C. 15 cm D. 7 cm

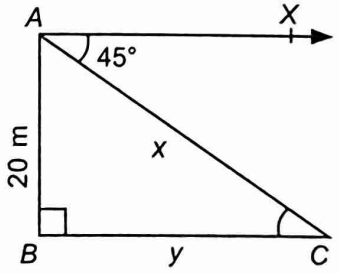
32. If $(x - 2)$ and $\left(x - \frac{1}{2}\right)$ both are factors of the polynomial $px^2 + 5x + r$, then p =

A. $\frac{3}{4}r$ B. $2r$ C. $\frac{r}{2}$ D. r

33. Find the length of a chord which is at a distance of 8 cm from the centre of the circle of radius 17 cm.

A. 24 cm B. 15 cm C. 30 cm D. 45 cm

34. In the given figure, if $\angle XAC = 45°$ = angle of depression, then the value of x and y are respectively



A. $20\sqrt{2}$ m, 20 m B. 20 m, $20\sqrt{2}$ m

C. $20\sqrt{3}$ m, 20 m D. 20 m, $20\sqrt{3}$ m

35. Solve for x and y :

$\frac{a}{x} - \frac{b}{y} = 0$ and

$\frac{ab^2}{x} + \frac{a^2b}{y} = a^2 + b^2$, where $x, y \neq 0$.

A. $x = \frac{a(a^2+b^2)}{a^2+b^2}, y = \frac{a^2-b^3}{-(a^2+b^2)}$

B. $x = \frac{a(a^2-b^2)}{a^2+b^2}, y = \frac{a^2b-b^3}{-(a^2+b^2)}$

C. $x = \frac{a^2+b^2}{a(a^2-b^2)}, y = \frac{-(a^2+b^2)}{a^2b-b^3}$

D. $x = a, y = b$

EVERYDAY MATHEMATICS

36. A factory produces on an average 4500 items per month for the first 4 months. How many items it must produce on an average per month over the next 8 months, to average 4800 items per month over the whole?

A. 5100 B. 4600 C. 4950 D. 4710

37. If we buy 2 tickets from station A to station B and 3 from station A to C, we have to pay ₹ 795. But 3 tickets from A to B and 5 tickets from A to C cost a total of ₹ 1300. What is the fare from A to B and from A to C respectively?

A. ₹ 85, ₹ 312 B. ₹ 75, ₹ 215

C. ₹ 65, ₹ 115 D. ₹ 95, ₹ 125

38. The top of two poles are connected by a 50 m wire which makes an angle of 60° with the horizontal, then find the distance between the bottoms of two poles.

A. 35 m B. 15 m C. 25 m D. 45 m

39. A car travels from city A to city B at a constant speed. If its speed be increased by 12 km/h, it would have taken one hour less to cover the distance. It would have taken further 45 minutes less if the speed is further increased by 12 km/h. What is the distance between the cities?

A. 540 km B. 504 km

C. 405 km D. 450 km

40. A person invested in all ₹ 2600 at 4 %, 6 % and 8 % per annum at simple interest. At the end of the year, he got the same interest in all the three cases. The money invested at 6 % is

A. ₹ 200 B. ₹ 600

C. ₹ 800 D. ₹ 1200

41. 10 women can complete a work in 7 days and 10 children take 14 days to complete the work. How many days will 5 women and 10 children take to complete the work?

A. 3 B. 5 C. 7 D. 9

42. A cube of side 3 cm weighs 12 kg. What is the weight of the similar cube of same material whose side length is 12 cm?

A. 768 kg B. 678 kg

C. 964 kg D. 864 kg

43. A solution of salt and water contains 42% salt by weight. If 25 kg of water evaporates and presently new solution contains 56% of salt, then find the original quantity of solution.

A. 120 kg B. 100 kg

C. 98 kg D. 123 kg

44. Two tanks are of the same capacity. The dimensions of the first tank are 15 cm × 12 cm × 8 cm. The second tank has a square base with depth 10 cm, then find the side of the square.

A. 12 cm B. 6 cm C. 8 cm D. 10 cm

45. Rohan's father arranges to pay off a debt of ₹ 3600 by 40 annual instalments which are in A.P. When 30 of the instalments were paid, his father died leaving one-third of the debt unpaid. The value of 10^{th} instalment is

A. ₹ 35 B. ₹ 59 C. ₹ 65 D. ₹ 69

ACHIEVERS SECTION

46. Study the following statements carefully and select the correct option.

Statement I : Both the roots of the equation $x^2 - x + 1 = 0$ are real.

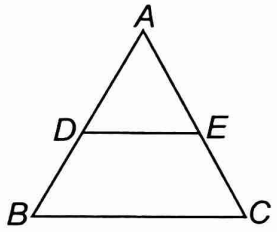
Statement II : The roots of the equation $x^2 - 5x + 5 = 0$ are real and distinct.

Which of the following options hold?

A. Both Statement I and Statement II are true.
B. Both Statement I and Statement II are false.
C. Statement I is true but Statement II is false.
D. Statement I is false but Statement II is true.

47. Fill in the blanks.

(i) In the given figure, $DE \| BC$. If $AD = x$ cm, $DB = (x - 2)$ cm, $AE = (x + 2)$ cm and $EC = (x - 1)$ cm, then the value of x is _____.



(ii) $ABCD$ is a trapezium such that $BC \| AD$ and $AD = 4$ cm. If the diagonals AC and BD intersect at O such that $\frac{AO}{OC} = \frac{DO}{OB} = \frac{1}{2}$, then $BC =$ _____.

(iii) In a right angle triangle KEY, sides are x cm, 40 cm, and 41 cm such that x cm is the shortest side, then $x =$ _____.

	(i)	(ii)	(iii)
A.	8 cm	7 cm	6 cm
B.	9 cm	10 cm	11 cm
C.	12 cm	13 cm	14 cm
D.	4 cm	8 cm	9 cm

48. V_1, V_2, V_3 and V_4 are the volumes of four cubes of side lengths x cm, $2x$ cm, $3x$ cm and $4x$ cm respectively. Some statements regarding these volumes are shown here.

(1) $V_1 + V_2 + 2V_3 < V_4$
(2) $V_1 + 4V_2 + V_3 < V_4$
(3) $2(V_1 + V_3) + V_2 = V_4$

Which of the given statements is correct?

A. (1) and (2) only B. (2) and (3) only
C. (1) and (3) only D. (1), (2) and (3)

49. In ΔABC, right angled at B and cosec $A = \sqrt{2}$, then state 'T' for 'true' and 'F' for 'false' and select the correct option.

(i) The value of $\frac{1}{\tan A} + \frac{\sin A}{1 + \cos A}$ is $\sqrt{2}$.

(ii) The value of $\sin C + \cos C + \sin B$ is $\sqrt{2} + 1$.

(iii) The value of $\sec^2 A - \text{cosec}^2 A + 2$ is 4.

	(i)	(ii)	(iii)
A.	F	T	F
B.	T	F	F
C.	T	F	T
D.	T	T	F

50. Which of the following steps of construction is incorrect while drawing a tangent to a circle of radius 6 cm and making an angle of 30° with a line passing through the centre?

Steps of construction :

Step-I : Draw a circle with centre *O* and radius 3 cm.

Step-II : Draw a radius *OA* of this circle and produce it to *B*.

Step-III : Construct an angle $\angle AOP$ equal to the complement of 30° *i.e.*, equal to 60°.

Step-IV : Draw perpendicular to *OP* from *A* which intersects *OA* produced at *Q*.

Clearly, *PQ* is the desired tangent such that $\angle OQP = 30°$.

A. Both step I and step IV
B. Only step III
C. Both step III and step IV
D. Only step I

☺☺☺

Darken your choice with HB Pencil

1.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	14.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	27.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	40.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
2.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	15.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	28.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	41.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
3.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	16.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	29.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	42.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
4.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	17.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	30.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	43.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
5.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	18.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	31.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	44.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
6.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	19.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	32.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	45.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
7.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	20.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	33.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	46.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
8.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	21.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	34.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	47.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
9.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	22.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	35.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	48.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
10.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	23.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	36.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	49.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
11.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	24.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	37.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	50.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
12.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	25.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	38.	Ⓐ	Ⓑ	Ⓒ	Ⓓ					
13.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	26.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	39.	Ⓐ	Ⓑ	Ⓒ	Ⓓ					