CHAPTER-2

POLYNOMIALS

KEY POINTS

1. A Polynomial p(x) in one variable x is an algebraic expression in x of the form $p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + ... + a_2 x^2 + a_1 x + a_0$, where

   (i) $a_0, a_1, a_2 ..... a_n$ are constants and $a_n \neq 0$

   (ii) $a_0, a_1, a_2 ..... a_n$ are respectively the coefficients of $x^0, x, x^2, ... x^n$,

   (iii) Each of $a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + ... + a_2 x^2 + a_1 x + a_0$, is called a term of the polynomial.

   (iv) n is called the degree of the polynomial where n is a non-negative integers.

2. **Degree of the Polynomial :** Highest power of x in the algebraic expression is called the degree of the polynomial.

3. **Different types of polynomials :**

   Generally, we divide the polynomials in the following categories :

   (i) Based on degrees

	Degree	Polynomial	General form	Examples
(a)	1	Linear	$ax + b$,	$x + 1, 2x$ etc.
(b)	2	Quadratic	$ax^2 + bx + c$,	$4x^2 + 5x + \frac{2}{3}$ etc.
(c)	3	Cubic	$ax^3 + bx^2 + cx + d$,	$x^3 - 3x^2 + 5$ etc.
(d)	4	Biquadratic	$ax^4 + bx^3 + cx^2 + dx + e$,	$x^4 - 16$ etc.

   a, b, c, d, e are real constants and $a \neq 0$.

   Note : A polynomial of degree five or more than five does not have any particular name. Such a polynomial usually called a polynomial of degree five or six or ... etc.

   (ii) Based on Number of Terms:

	No. of Terms	**Polynominal**	**Examples**
(a)	1	Monomial	$5, 3x, \frac{1}{3}y$ etc.
(b)	2	Binomial	$\sqrt{3} + 6x, x - 5y, x^2 + 2$ etc.
(c)	3	Trinomial	$\sqrt{2}x^2 + 4x + 2, 5y^4 + 2y + 6$ etc.

Note : A polynomial having four or more than four terms does not have particular name. These are simply called polynomials.

(iii) Zero degree polynomial or non-zero constant polynomial.

Any non-zero number (constant) is regarded as polynomial of degree zero or zero degree polynomial. i.e., $p(x) = a$ where $a \neq 0$ is a zero degree polynomial, since we can write $p(x) = a$,

as $p(x) = ax^0$

e.g., $5 = 5x^0$, $\frac{\sqrt{7}}{2} = \frac{\sqrt{7}}{2}x^0$

(iv) Zero Polynomial : A polynomial whose all coefficients are zero is called as zero polynomial i.e., $p(x) = 0$. The degree of zero polynomial is not defined or we can not determine the degree of zero polynomial.

4. For a polynomial $p(x)$ if $p(a) = 0$ where a is a real number we say that 'a' is a zero of the polynomial.

5. If $p(x)$ is any polynomial of degree greater than or equal to 1 and $p(x)$ is divided by a linear polynomial $x - a$, then the remainder is $p(a)$. This is called remainder theorem.

6. If $p(x)$ is a polynomial of degree ≥ 1 and 'a' is any real number then

(i) $(x - a)$ is a factor of $p(x)$, if $p(a) = 0$ and

(ii) $p(a) = 0$ if $(x - a)$ is a factor of $p(x)$.

This is called factor theorem.

7. A polynomial of degree 'n' can have at most n zeroes.

- Some algebraic identities :–

(i) $(x+y)^2 = x^2 + 2xy + y^2$

(ii) $(x-y)^2 = x^2 - 2xy + y^2$

(iii) $x^2-y^2 = (x+y)(x-y)$

(iv) $(x+a)(x+b) = x^2 + (a+b)x + ab$

(v) $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(vi) $(x+y)^3 = x^3 + y^3 + 3xy\,(x+y) = x^3 + y^3 + 3x^2y + 3xy^2$

(vii) $(x-y)^3 = x^3 - y^3 - 3xy\,(x-y) = x^3 - y^3 - 3x^2y + 3xy^2$

(viii) $x^3+y^3 = (x+y)\,(x^2 - xy + y^2)$

ix) $x^3-y^3 = (x-y)\,(x^2+xy + y^2)$

x) $x^3 + y^3 + z^3 - 3xyz = (x+y+z)\,(x^2+y^2+z^2-xy-yz-zx)$

$$= \frac{1}{2}\,(x+y+z)\left\{(x-y)^2 + (y-z)^2 + (z-x)^2\right\}$$

xi) If $x+y+z = 0$, then $x^3 + y^3 + z^3 = 3xyz$

Part-A

1. Write the coefficient of y^3 in $5y^3 + 2y^2 - y + 5$

2. Find the coefficient of x^2 in $(x^2 - 1)(x - 2)$

3. If $(x - 2)$ is one of the factor of $3x - 2a$, then find the value of a.

4. Find the degree of polynomial $\frac{x^3 + 3x - 1}{5} - \frac{5}{2}x^2 - x^5$

5. If $p(x) = x^3 - 3x^2 + 2x - 3$ find the value of $p(1) + p(-1)$.

6. Find zeros of the polynomial $z^2 - 8$

7. Dividend = Divisor × Quotient + _____.

8. Give an example of Trinomial of degree 3.

9. Give one example each of monomial, binomial and quadratic polynomial.

10. Check whether $x = 3$ is a zero of polynomial $x^2 - 3x + x - 3$.

11. Write the degree of the polynomial $\sqrt{7}$

12. If one of the zero of polynomial $3x^2 + 5x + k$ is -1, then find out the value of k.

13. Express $4x^2 - 4x + 1$ as a square of binomial.

Part – B

14. Check whether q(x) is a multiple of r(x) or not.

    If $q(x) = 2x^3 - 11x^2 - 4x + 5$, $r(x) = 2x + 1$

15. Show that $(x - 5)$ is a factor of $x^3 - 3x^2 - 4x - 30$.

16. Evaluate by using suitable identity : $(997)^3$

17. Find the zeroes of the polynomial $p(x) = x(x-2)(x+3)$

18. Find the quotient when $3x^2 - 7x - 6$ is divided by $(x-3)$

19. Factorise $8x^3 + \sqrt{27}\,y^3$.

20. If $p(x) = x + 9$, then find $p(x) + p(-x)$.

21. Find the product without multiplying directly

    106×94

22. IF $36x^2 - b = \left(6x + \frac{1}{5}\right)\left(6x - \frac{1}{5}\right)$ then find the value of b.

23. Expand using suitable identity $(2x - 3y + z)^2$

24. Find the value of $(351)^2 - (350)^2$.

Part – C

25. Factorise : $64a^2 + 96ab + 36b^2$

26. Factorise : $x^3 + 6x^2 + 11x + 6$

27. If $x^2 + y^2 = 49$ and $x - y = 3$, then find the value of $x^3 - y^3$.

28. Simplify : $(5a - 2b)(25a^2 + 10ab + 4b^2) - (2a + 5b)(4a^2 - 10ab + 25b^2)$

29. Find the sum of remainders when $x^3 - 3x^2 + 4x - 4$ is divided by $(x - 1)$ and $(x + 2)$.

30. Find the product $\left(p - \frac{1}{p}\right)\left(p + \frac{1}{p}\right)\left(p^2 + \frac{1}{p^2}\right)\left(p^4 + \frac{1}{p^4}\right)$

31. Factorise : $7\sqrt{2}\,k^2 - 10k - 4\sqrt{2}$.

32. Simplify : $(3x - 4y)^3 - (3x + 4y)^3$

33. Expand : $\left(\frac{1}{2}x - \frac{1}{4}y + 2\right)^2$ using suitable identity.

34. Simplify : $(x + y + z)^2 - (x - y - z)^2$.

Part – D

35. Factorise : $125x^3 + 8y^3 + z^3 - 30xyz$.

36. $x + 2$ is a factor of polynomial $ax^3 + bx^2 + x - 2$ and the remainder 4 is obtained by dividing this polynomial by $(x - 2)$. Find the value of a and b.

37. Check whether

    $p(t) = 6t^3 + 3t^2 + 3t + 18$ is a multiple of $(2t + 3)$.

38. Find the value of k if $(x + k)$ is a factor of the polynomial $x^3 + kx^2 - 2x + k + 4$ and factorise $x^4 - x$.

39. If $(x - 3)$ and $\left(x - \frac{1}{3}\right)$ are factors of the polynomial $px^2 + 3x + r$, show that $p = r$.

40. (i) Using Identity, find the value of $(-7)^3 + (5)^3 + (2)^3$.

    (ii) Find dimensions of cube whose volume is given by expression $4x^2 + 14x + 6$

41. Give possible expression for the length and breadth of each of the following rectangles if.

    (i) Area = $(x^2 + 5\sqrt{5}x + 30)$ sq. unit.

    (ii) Area = $(24x^2 - 26x - 8)$ sq. unit.

42. A literacy compaign was organised by Class IX girl students under NSS. Students made $(x - 5)$ rows and $(3x - 4)$ columns for the rally.

    Write the total number of students in the form of a polynomial.

43. Under tree plantation programme students of Class IX planted total $(3x^2 - 4x - 4)$ trees in school.

    If total number of students in the class are $(x - 2)$ then find out number of trees planted by each student. (Assuming each student planted equal number of trees).

44. If $a + b + c = 0$, find the value of

$$\frac{(b+c)^2}{bc} + \frac{(c+a)^2}{ca} + \frac{(a+b)^2}{ab}$$

45. Simplify :

$$\frac{(a^2-b^2)^3 + (b^2-c^2)^3 + (c^2-a^2)^3}{(a-b)^3 + (b-c)^3 + (c-a)^3}$$

46. Factorise :

$$(2a-b-c)^3 + (2b-c-a)^3 + (2c-a-b)^3$$

47. If the polynomial $4x^3-16x^2+ax+7$ is exactly divisible by $x-1$, then find the value of a. Hence factorise the polynomial.

48. Factorise : $x^2 - \frac{13}{24}x - \frac{1}{12}$

49. Factorise : $9x^3 - 27x^2 - 100x + 300$

50. Factorise : $x^4 - 5x^2 + 4$

51. If $\frac{x}{y} + \frac{y}{x} = -1$ where $x \neq 0$, $y \neq 0$ then find the value of $x^3 - y^3$.

52. Simplify: $\frac{155 \times 155 + 155 \times 55 + 55 \times 55}{155 \times 155 \times 155 - 55 \times 55 \times 55}$

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ANSWERS

1. 5 2. –2 3. $a = 3$

4. 5 5. –12 6. $+\sqrt{8}, -\sqrt{8}$

7. Remainder 8. $x^3 - 3x^2 + 2$ or any other example

9. $2x, 2x^2 + 3, x^2 + 2x - 3$ or any other examples

10. Yes 11. Degree = 0 12. $k = 2$

13. $(2x-1)^2$ 14. No. 15. Hint put $x = 5$

16. 991026973 17. 0, 2, –3 18. $3x + 2$

19. $\left(2x + \sqrt{3}y\right)\left(4x^2 - 2\sqrt{3}xy + 3y^2\right)$ 20. 18

21. Hint $(100 + 6)(100 - 6)$ 22. $\frac{1}{25}$

23. $4x^2 + 9y^2 + z^2 - 12xy - 6yz + 4xz$ 24. 701

25. $(8a + 6b)^2$ 26. $(x + 1)(x + 2)(x + 3)$

27. 207 28. $117a^3 - 133b^3$ 29. –34

30. $p^8 - \frac{1}{p^8}$ 31. $\left(k - \sqrt{2}\right)\left(7\sqrt{2}k + 4\right)$

32. $-8y(16y^2 + 27x^2)$ or $-128y^3 - 216x^2y$

33. $\frac{x^2}{4} + \frac{y^2}{16} + 4 - \frac{1}{4}xy - y + 2x$ 34. $4xy + 4zx$

35. $(5x + 2y + z)(25x^2 + 4y^2 + z^2 - 10xy - 2yz - 5zx)$

36. $a = 0, b = 2$ 37. Yes

38. $k = \frac{4}{3}$, $x(x-1)\left(x^2 + x + 1\right)$

40. (i) –210; (ii) 2, $(x + 3)$, $(2x + 1)$

41. (i) $(x + 2\sqrt{5}), (x + 3\sqrt{5})$ (ii) $(4x + 1), (6x - 8)$

42. $3x^2-19x+20$

43. $(3x + 2)$

44. 3

45. $(a+b)(b+c)(c+a)$

46. $3(2a-b-c)(2b-c-a)(2c-a-b)$

47. $a= 5, (x-1)(2x+1)(2x-7)$

48. $\frac{1}{24}(3x-2)\ (8x+1)$

49. $(3x + 10)(x-3)(3x-10)$

50. $(x-1)(x+1)(x-2)(x+2)$

51. 0

52. 0.01

Practice Test
POLYNOMIALS

Time : 50 Min. M.M. 20

1. Is $(x^2)^{½} + 2\sqrt{5}$ a polynomial? (1)

2. Show that x = 1 is a zero of the polynomial $3x^3 – 4x^2 + 8x – 7$. (1)

3. Find the zeroes of the polynomial $x^2 – 4x + 3$ (2)

4. If x + y + z = 6, xy + yz + zx = 11. Find the value of $x^2 + y^2 + z^2$. (2)

5. If 3x – 4 is a factor of the polynomial $p(x) = 2x^3 – 11x^2 + kx – 20$, find the value k (3)

6. Factorise : $a^2 + b^2 + 2(ab + bc + ca)$ (3)

7. If a + b + c = 0 then find the value of (4)
$$\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$$

8. Factorise $x^3 – 23x^2 + 142x – 120$ by using factor theorem. (4)