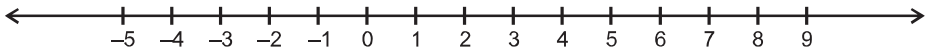
CHAPTER-1

NUMBER SYSTEMS

KEY POINTS



- 1, 2, 3, ... are natural numbers which are represented by N.
- 0, 1, 2, 3, ... are whole numbers which are represented by W.
- ..... –3, –2, –1, 0, 1, 2, 3, ... are Integers which are represented by Z or I.
- A number is rational number if
  (a) it can be represented in the form of p/q where p and q are integers and $q \neq 0$.

  or

  (b) its decimal expansion is terminating (e.g. $\frac{2}{5} = 0.4$)

  or

  (c) its decimal expansion is non-terminating recurring (repeating) (e.g. $0.\overline{1234} = 0.1\dot{2}3\dot{4} = 0.1234234.....$)
- A number is irrational number if
  (a) it can not be represented in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

  or

  (b) its decimal expansion is non-terminating non-recurring (e.g. 0.1010010001.....)
- All rational and irrational numbers collectively form real numbers.
- There are infinite rational numbers between any two rational numbers.
- There is a unique real number corresponding to every point on the number line. Also, corresponding to each real number, there is a unique point on the number line.
- Rationalisation of a denominator means to change the Irrational denominator to rational form.
- To rationalise the denominator of $\frac{1}{\sqrt{a} + b}$, We multiply this by $\frac{\sqrt{a} - b}{\sqrt{a} - b}$, where a and b are integers.

- Laws of Exponents : Let $a > 0$ be a real number and m and n are rational numbers, then

  1) $a^m \, a^n = a^{m+n}$ 2) $a^m \div a^n = a^{m-n}$

  3) $(a^m)^n = a^{mn}$ 4) $a^m . b^m = (ab)^m$

  5) $a^0 = 1$ 6) $a^{-m} = \frac{1}{a^m}$

- For positive real number a and b, the following Identities hold

  1) $\sqrt{a} \, . \, \sqrt{b} = \sqrt{ab}$ 2) $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$

  3) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$ 4) $(\sqrt{a} + \sqrt{b})^2 = a + 2\sqrt{ab} + b$

  5) $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$

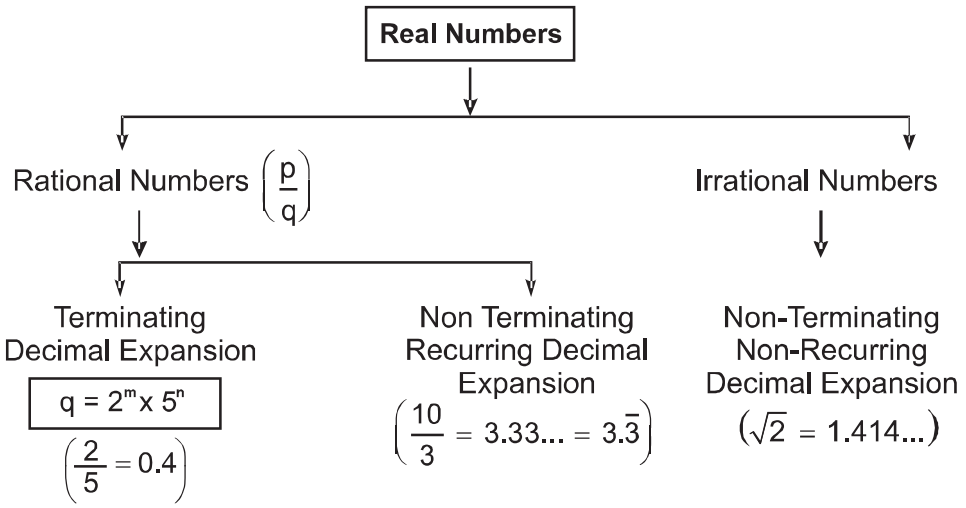
  All natural numbers, whole numbers and integers are rational

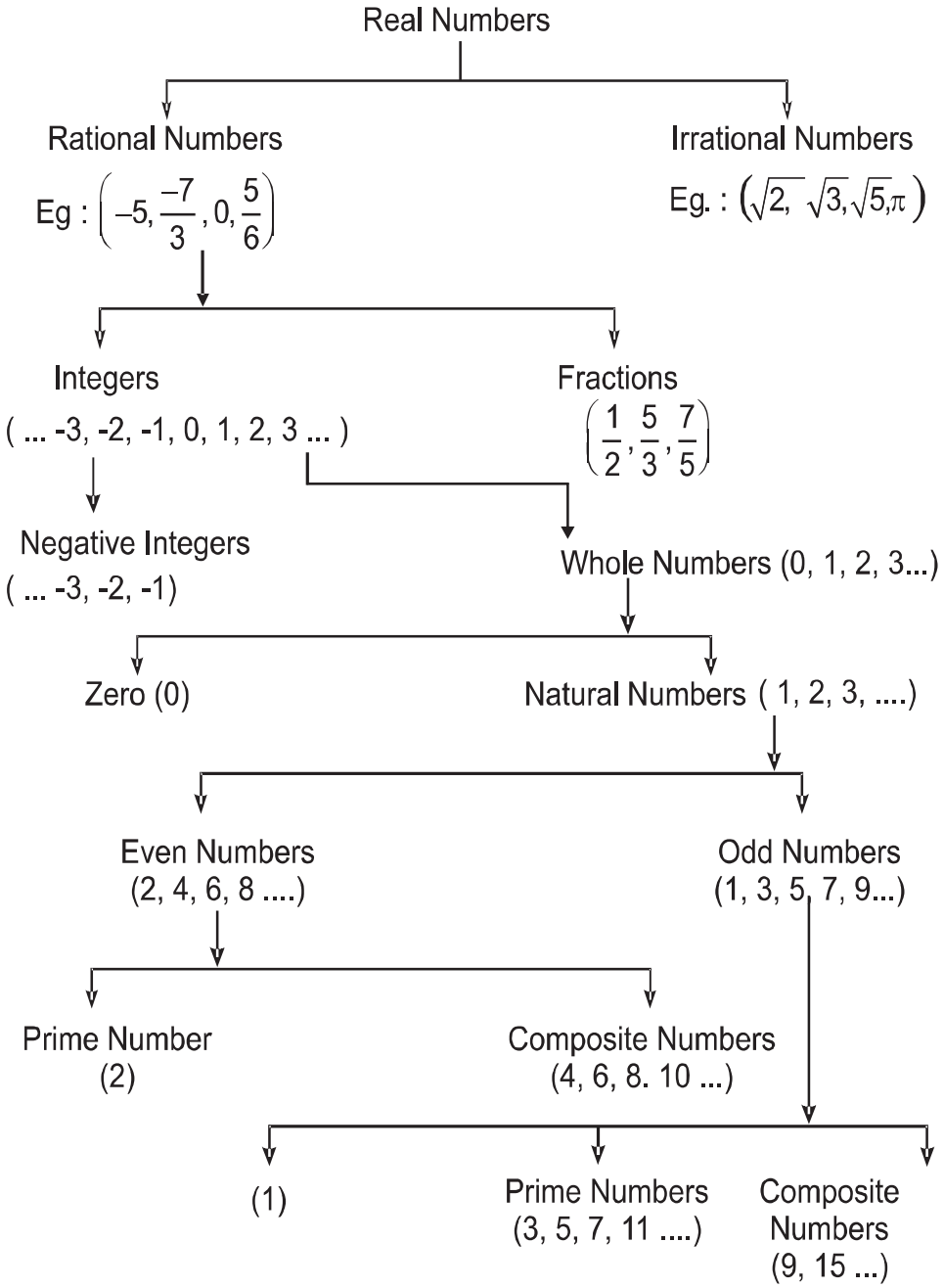
  Prime Numbers : All natural numbers that have exactly two factors (i.e., 1 and itself) are called prime numbers. e.g., 2, 3, 5, 7, 11, 13, 17, 19, 23, … etc.

  Composite Numbers : Those natural numbers which have more than two factors are known as composite numbers. e.g., 4, 6, 8, 10, 12, …

  1 is neither prime nor composite.

Types of Numbers




Real Numbers
Rational Numbers
Eg : $\left(-5, \frac{-7}{3}, 0, \frac{5}{6}\right)$
Irrational Numbers
Eg. : $\left(\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi\right)$
Integers
(... -3, -2, -1, 0, 1, 2, 3 ...)
Fractions
$\left(\frac{1}{2}, \frac{5}{3}, \frac{7}{5}\right)$
Negative Integers
(... -3, -2, -1)
Whole Numbers (0, 1, 2, 3...)
Zero (0)
Natural Numbers (1, 2, 3, ...)
Even Numbers
(2, 4, 6, 8 ....)
Odd Numbers
(1, 3, 5, 7, 9...)
Prime Number
(2)
Composite Numbers
(4, 6, 8. 10 ...)
(1)
Prime Numbers
(3, 5, 7, 11 ....)
Composite Numbers
(9, 15 ...)

* $\sqrt[n]{a} = a^{1/n}$

where 'a' is a positive real number and n is a positive integer.

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

where 'a' is a positive real number, m and n are co prime integers, and $n > 0$.

Part (A)

1. Write first five whole numbers in $\frac{p}{q}$ form, where p and q are integers and $q \neq 0$
2. Find decimal expansion of $\frac{17}{8}, \frac{3}{15}, \frac{2}{7}, \frac{50}{3}$.
3. Find four rational numbers between $\frac{2}{9}$ and $\frac{3}{7}$.
4. Find decimal form of $\sqrt{23}$ and $\sqrt{24}$ upto 3 decimal places.
5. Find two Irrational numbers between $\sqrt{23}$ and $\sqrt{24}$.
6. Find one Irrational and one rational number between 2 and $\sqrt{5}$.
7. Write two numbers whose decimal expansions are terminating.
8. What can be the maximum number of digits in the repeating block of digits in the decimal expansion of $\frac{5}{7}$?
9. Write two numbers whose decimal expansions are non-terminating non-repeating (non-recurring).
10. Find the value of $(256)^{0.16} \times (256)^{0.09}$
11. Find two Irrational numbers between 2016 and 2017.

Part (B)

12. Represent $\frac{-7}{5}$ on the number line.
13. Represent following on number line

    i) $\sqrt{5}$ ii) $\sqrt{13}$ iii) $\sqrt{9.3}$ iv) $\sqrt{2}$
14. Represent $3 + \sqrt{2.6}$ on the number line.
15. Insert two Irrational numbers between $\frac{2}{3}$ and $\frac{3}{2}$
16. Simplify : $\frac{\sqrt{5} + \sqrt{3}}{\sqrt{80} + \sqrt{48} - \sqrt{45} - \sqrt{27}}$
17. Find the value of $[1^3 + 2^3 + 3^3 + 8^2]^{-5/2}$
18. Find the value of x if $x^{1/2} = (36)^{0.5}$
19. Find the value of x if $(\sqrt{3})^x = 3^7$

20. If $2^{5x} \div 2^{x} = \sqrt[5]{32}$. Then find the value of x.

21. Evaluate $a^{x-y}.a^{y-z}.a^{z-x}$.

22. Simplify $12^{\frac{2}{5}} . 5^{\frac{2}{5}}$.

23. Which of the following rational numbers will have a terminating decimal expansion or a non-terminating repeating (recurring) decimal expansion ?

(i) $\frac{135}{50}$ (ii) $\frac{4}{11}$ (iii) $\frac{8}{7}$ (iv) $6\frac{3}{8}$

(v) $\frac{55}{9}$ (vi) $\frac{5^2 \times 3^3}{2 \times 5^3 \times 27}$ (vii) $\frac{51}{60}$.

24. Classify the following numbers as terminating decimal or non-terminating recurring decimal or non-terminating non-recurring decimal :

(i) 0.1666... (ii) 0.250 (iii) 1.01001000100001....

(iv) 0.27696 (v) 2.142857142857.... (vi) $0.\overline{3}$

(vii) 0.2359872785... (viii) 0.484848848.... (ix) 2.502500250002.....

(x) $4.\overline{123456789}$

Also classify these given numbers as Rational and Irrational numbers.

25. Classify the following numbers as rational or Irrational number :

(i) $\sqrt{27}$ (ii) $\sqrt{36}$ (iii) $\sqrt{5} \times \sqrt{125}$ (iv) $2\sqrt{3}$

(v) $\frac{7\sqrt{7}}{\sqrt{343}}$ (vi) $2 + \sqrt{21}$ (vii) $5 + 2\sqrt{23} - (\sqrt{25} + \sqrt{92})$

(viii) $\frac{22}{7}$ (ix) π (x) $\sqrt[3]{27}$

26. Express the following numbers in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(i) 0.0875 (ii) 2.123456789 (iii) 0.181818.....

(iv) $0.4\overline{37}$ (v) $3.65\overline{1}$

27. Do as directed :

(i) Add : $\sqrt{125} + 2\sqrt{27}$ and $-5\sqrt{5} - \sqrt{3}$

(ii) Add : $\sqrt{7} - \sqrt{11}$ and $\sqrt{5} - \sqrt{11} + \sqrt{13}$

(iii) Multiply : $2\sqrt{2}$ by $5\sqrt{2}$.

(iv) Multiply : $(-3 + \sqrt{5})$ by 3.

(v) Divide : $7\sqrt{5}$ by $-14\sqrt{125}$

(vi) Divide : $2\sqrt{216} - 3\sqrt{27}$ by 3.

Part (C)

28. Simplify :

(i) $(2\sqrt{2} + 3\sqrt{3})(2\sqrt{2} - 3\sqrt{3})$ (ii) $(2\sqrt{8} - 3\sqrt{2})^2$

(iii) $(\sqrt{7} + \sqrt{6})^2$ (iv) $(6 - \sqrt{2})(2 + \sqrt{3})$

29. Evaluate :

(i) $\dfrac{2^{38} + 2^{37} + 2^{36}}{2^{39} + 2^{38} + 2^{37}}$ (ii) $\left[\left(64^{\frac{1}{2}}\right)^{\frac{1}{6}}\right]^2$

30. Find the value of a if $\dfrac{6}{3\sqrt{2} - 2\sqrt{3}} = 3\sqrt{2} - a\sqrt{3}$.

31. Simplify : $\left[5\left(8^{1/3} + 27^{1/3}\right)^3\right]^{\frac{1}{4}}$

32. Simplify : $\dfrac{(25)^{3/2} \times (243)^{3/5}}{(16)^{5/4} \times (8)^{4/3}}$

33. If $5^{2x-1} - (25)^{x-1} = 2500$, then find the value of x.

Part (D)

34. Express $0.6 + 0.\overline{7} + 0.4\overline{7}$ in the form $\dfrac{p}{q}$ where p and q are integers and $q \neq 0$.

35. Rationalise the denominator of $\frac{1}{\sqrt{3}+\sqrt{5}+\sqrt{7}}$

36. Find *a* and *b* if $\frac{7+3\sqrt{5}}{2+\sqrt{5}} - \frac{7-3\sqrt{5}}{2-\sqrt{5}} = a + b\sqrt{5}$

37. If $x = (3 - 2\sqrt{2})$, show that $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) = \pm 2$

38. If $xyz = 1$, then simplify

$$(1 + x + y^{-1})^{-1} \times (1 + y + z^{-1})^{-1} \times (1 + z + x^{-1})^{-1}$$

39. Find the value of *x* if

(i) $25^{2x-3} = 5^{2x+3}$ (ii) $(4)^{2x-1} - (16)^{x-1} = 384$

40. Evaluate : $\frac{64^{\frac{a}{6}}}{4^a} \times \frac{2^{2a+1}}{2^{a-1}}$

41. Simplify : $\frac{1}{1 + x^{b-a} + x^{c-a}} + \frac{1}{1 + x^{a-b} + x^{c-b}} + \frac{1}{1 + x^{a-c} + x^{b-c}}$

42. Simplify : $\left(\frac{x^a}{x^{-b}}\right)^{a-b} \times \left(\frac{x^b}{x^{-c}}\right)^{b-c} \times \left(\frac{x^c}{x^{-a}}\right)^{c-a}$

43. Show that :

$$\frac{1}{(3-\sqrt{8})} - \frac{1}{(\sqrt{8}-\sqrt{7})} + \frac{1}{(\sqrt{7}-\sqrt{6})} - \frac{1}{(\sqrt{6}-\sqrt{5})} + \frac{1}{(\sqrt{5}-2)} = 5$$

44. If $a = \frac{\sqrt{7}-\sqrt{6}}{\sqrt{7}+\sqrt{6}}$ and $b = \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}-\sqrt{6}}$, then find the value of $a^2 + b^2 + ab$.

45. Simplify : $\frac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}} + \frac{6\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{8\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

46. If $x = 9 - 4\sqrt{5}$, then find

(i) $x + \frac{1}{x}$ (ii) $x - \frac{1}{x}$ (iii) $x^2 + \frac{1}{x^2}$ (iv) $x^2 - \frac{1}{x^2}$

(v) $x^3 + \frac{1}{x^3}$ (vi) $x^3 - \frac{1}{x^3}$ (vii) $\sqrt{x} + \frac{1}{\sqrt{x}}$ (viii) $\sqrt{x} - \frac{1}{\sqrt{x}}$

(ix) $x^4 + \frac{1}{x^4}$ (x) $x^6 + \frac{1}{x^6}$ (xi) $x + \frac{14}{x}$

47. If $a = 1 + \sqrt{7}$, find the value of $\frac{-6}{a}$

48. If $p = 5 - 2\sqrt{6}$, Find $p^2 + \frac{1}{p^2}$

49. Express $0.3\overline{178}$ in the form of p/q where p and q are integers and $q \neq 0$.

50. If $\sqrt{2} = 1.414$, then find the value of $\sqrt{8} + \sqrt{50} + \sqrt{72} + \sqrt{98}$

51. Find the value of

$$\frac{4}{(216)^{\frac{-2}{3}}} + \frac{1}{(256)^{\frac{-3}{4}}} + \frac{2}{(243)^{\frac{-1}{5}}}$$

CHAPTER-1

NUMBER SYSTEMS

ANSWERS

1) $\frac{0}{1}, \frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}$

2) $\frac{17}{8} = 2.125, \frac{3}{15} = 0.2, \frac{2}{7} = 0.\overline{285714}, \frac{50}{3} = 16.\bar{6}$

3) $\frac{15}{63}, \frac{16}{63}, \frac{17}{63}, \frac{18}{63}$ (other answers are possible).

4) $\sqrt{23} = 4.795, \sqrt{24} = 4.898$

5) 4.8010010001 ....., 4.8020020002 ....., (other answers are possible)

6) 2.1, 2.010010001 ....., (other answers are possible).

8) 6

10) 4

11) 2016.1010010001 ..... ; 2016.2020020002 .....; (other answers are possible)

15) 0.909009000 .....; 1.10100100010000 ..... (other answers are possible)

16) 1 17) $\frac{1}{10^5}$ 18) 36 19) 14

20) $x = \frac{1}{4}$ 21) 1 22) $(60)^{2/5}$

23) (i) Terminating Decimal (ii) Non Terminating Repeating Decimal

(iii) Non-Terminating Repeating Decimal

(iv) Terminating Decimal (v) Non-Terminating Repeating Decimal

(vi) Terminating Decimal (vii) Terminating Decimal

24) (i) Non-Terminating Repeating Decimal (Rational).

(ii) Terminating Decimal (Rational).

(iii) Non-Terminating Non-Repeating Decimal (Irrational).

(iv) Terminating Decimal (Rational)

(v) Non-Terminating Repeating Decimal (Rational)

(vi) Non-Terminating Repeating Decimal (Rational)

(vii) Non-Terminating Non-Repeating Decimal (Irrational)

(viii) Non-Terminating Non-Repeating Decimal (Irrational)

(ix) Non-Terminating Non-Repeating Decimal (Irrational)

(x) Non-Terminating Repeating Decimal (Rational).

25. (i) Irrational (ii) Rational (iii) Rational (iv) Irrational

(v) Rational (vi) Irrational (vii) Rational (viii) Rational

(ix) Irrational (x) Rational

26. (i) $0.0875 = \frac{7}{80}$ (ii) $\frac{2123456789}{1000000000}$ (iii) $\frac{2}{11}$

(iv) $\frac{433}{990}$ (v) $\frac{1643}{450}$

27. (i) $5\sqrt{3}$ (ii) $\sqrt{5} - 2\sqrt{11} + \sqrt{7} + \sqrt{13}$ (iii) 20

(iv) $-9 + 3\sqrt{5}$ (v) $-\frac{1}{10}$ (vi) $4\sqrt{6} - 3\sqrt{3}$

28. (i) –19 (ii) 2 (iii) $13 + 2\sqrt{42}$

(iv) $12 + 6\sqrt{3} - 2\sqrt{2} - \sqrt{6}$

29. (i) $\frac{1}{2}$ (ii) 2

30. $a = -2$ 31. 5 32. $\frac{3375}{512}$

33. $x = 3$ 34. $\frac{167}{90}$

35. $\frac{1}{59}\left(9\sqrt{3}+5\sqrt{5}+\sqrt{7}-2\sqrt{105}\right)$

36. $a=0, b=2$

38. $\frac{1}{(1+y+xy)(1+z+yz)(1+x+zx)}$

39. (i) $\frac{9}{2}$ (ii) $\frac{11}{4}$

40. 4 41. 1 42. 1

44. $a^2+b^2+ab=675$

45. 0

46. (i) 18 (ii) $-8\sqrt{5}$ (iii) 322

(iv) $-144\sqrt{5}$ (v) 5778 (vi) $-2584\sqrt{5}$

(vii) $2\sqrt{5}$ (viii) 4 (ix) 103682

(x) 33385282 (xi) $8\sqrt{3}-14\sqrt{2}$

47. $1-\sqrt{7}$ 48. 98 49. $\frac{635}{1998}$

50. 28.28 51. 214

Practice Test
NUMBER SYSTEMS

Time : 50 Min. M.M. 20

1. If $\frac{4}{a} = \frac{a^2}{16}$, then find a is rational or irrational number. (1)

2. Find two irrational numbers between $\sqrt{2}$ and $\sqrt{3}$. (1)

3. Simplify:

   $4\sqrt{3} + 3\sqrt{48} - \frac{5}{2}\sqrt{\frac{4}{3}}$ (2)

4. If $\sqrt{3} = 1.732$, find the value of $\frac{2}{\sqrt{3}-1}$ (2)

5. Find the value of x and y (3)

   $$\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = a - b\sqrt{77}$$

6. Represent $(2+\sqrt{3})$ on the number line. (3)

7. Simplify : (4)

   $$\frac{16 \times 2^{a+1} - 4 \times 2^{a}}{16 \times 2^{a+2} - 2 \times 2^{a+2}}$$

8. Express the following in the form p/q where p and q are integers and $q \neq 0$ (4)

   $0.\overline{4} + 0.1\overline{8}$