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Chapter

Introduction to Trigonometry

Key Points

1. Trigonometric ratio : In $\triangle ABC$, $\angle B = 90°$. For $\angle A$,

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$
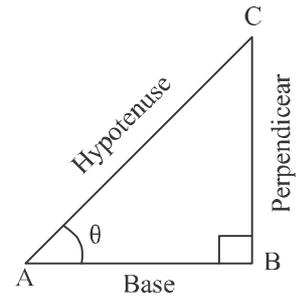
$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\text{adjacent side}}{\text{Hypotenuse}}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{\text{Opposite side}}{\text{adjacent side}}$$

$$\cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{\text{adjacent side}}{\text{opposite side}}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{\text{Hypotenuse}}{\text{adjacent side}}$$

$$\operatorname{cosec} A = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{\text{Hypotenuse}}{\text{Opposite side}}$$

2. Opposites

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta}, \ \operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}, \ \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}, \ \cot\theta = \frac{1}{\tan\theta}$$

3. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}, \ \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Mathematics-X

4. **Identities**

$\sin^2 \theta + \cos^2 = 1 \Rightarrow \sin^2 \theta = 1 - \cos^2 \theta$ and $\cos^2 \theta = 1 - \sin^2 \theta$

$1 + \tan^2 \theta = \sec^2 \theta \Rightarrow \tan^2 \theta = \sec^2 \theta - 1$ and $\sec^2 \theta - \tan^2 \theta = 1$

$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta \Rightarrow \cot^2 \theta = \operatorname{cosec}^2 \theta - 1$ and $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$

5. **Trigonometric ratios of some specific angles**

$\angle A$	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin A$	0	$\dfrac{1}{2}$	$\dfrac{1}{\sqrt{2}}$	$\dfrac{\sqrt{3}}{2}$	1
$\cos A$	1	$\dfrac{\sqrt{3}}{2}$	$\dfrac{1}{\sqrt{2}}$	$\dfrac{1}{2}$	0
$\tan A$	0	$\dfrac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\cot A$	Not defined	$\sqrt{3}$	1	$\dfrac{1}{\sqrt{3}}$	0
$\sec A$	1	$\dfrac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\operatorname{cosec} A$	Not defined	2	$\sqrt{2}$	$\dfrac{2}{\sqrt{3}}$	1

6. **Trigonometric ratios of complimentary angles**

$\sin(90° - \theta) \quad = \quad \cos \theta$

$\cos(90° - \theta) \quad = \quad \operatorname{Sin} \theta$

$\tan(90° - \theta) \quad = \quad \cot \theta$

$\cot(90° - \theta) \quad = \quad \tan \theta$

$\sec(90° - \theta) \quad = \quad \operatorname{cosec} \theta$

$\operatorname{cosec}(90° - \theta) \quad = \quad \sec \theta$

VERY SHORT ANSWER TYPE QUESTIONS

1. If $\operatorname{Sin} \theta = \cos \theta$, find the value of θ

2. If $\tan \theta = \cot(30° + \theta)$, find the value of θ

3. If $\operatorname{Sin} \theta = \cos(\theta - 6°)$, find the value of θ

4. If $\cos A = \dfrac{7}{25}$, find the value of $\tan A + \cot A$

5. If $\tan \theta = \dfrac{4}{3}$ then find the value of $\dfrac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta}$

6. If $3x = \csc \theta$ and $\dfrac{3}{x} = \cot \theta$ then find $3\left(x^2 - \dfrac{1}{x^2}\right)$

7. If $x = a \sin \theta$ and $y = a \cos \theta$ then find the value of $x^2 + y^2$

8. Find the value of $\csc 70° - \sec 20°$

9. If $5x = \sec \theta$ and $\dfrac{5}{x} = \tan \theta$ then find the value of $5\left(x^2 - \dfrac{1}{x^2}\right)$

10. Find the value of $9 \sec^2 A - 9 \tan^2 A$

11. Express $\sec \theta$ in terms of $\cot \theta$

12. Find the value of $\cos \theta \cos (90° - \theta) - \sin \theta \sin (90° - \theta)$

13. If $\sin (20° + \theta) = \cos 30°$ then find the value of θ.

14. Find the value of $\dfrac{1 + \tan^2 \theta}{1 + \cot^2 \theta}$

15. Find the value of $\dfrac{\sin \theta}{\sqrt{1 - \sin^2 \theta}}$

SHORT ANSWER TYPE (I) QUESTIONS

Prove that :

16. $\sec^4 \theta - \sec^2 \theta = \tan^4 \theta + \tan^2 \theta$

17. $\sqrt{\dfrac{1 + \sin \theta}{1 - \sin \theta}} = \tan \theta + \operatorname{Sec} \theta$

18. If $x = p \sec \theta + q \tan \theta$ & $y = p \tan \theta + q \sec \theta$ then prove that $x^2 - y^2 = p^2 - q^2$

19. If $7 \sin^2 \theta + 3 \cos^2 \theta = 4$ then show that $\tan \theta = \dfrac{1}{\sqrt{3}}$

Mathematics-X

20. If $\sin(A-B) = \dfrac{1}{2}$, $\cos(A+B) = \dfrac{1}{2}$ then find the value of A and B.

21. Find the value of $\dfrac{\cos^2 20° + \cos^2 70°}{\sin^2 59° + \sin^2 31°}$.

22. Prove that : $\tan 1° \tan 11° \tan 21° \tan 69° \tan 79° \tan 89° = 1$

23. If $\sec 4A = \operatorname{cosec}(A-20°)$ then find the value of A.

24. If $3 \cot A = 4$, find the value of $\dfrac{\operatorname{Cosec}^2 A + 1}{\operatorname{Cosec}^2 A - 1}$.

25. If $\tan(3x - 15°) = 1$ then find the value of x.

SHORT ANSWER TYPE QUESTIONS

Prove that :

26. $\dfrac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \dfrac{1 + \sin A}{\cos A}$

27. $\dfrac{1}{\sec x - \tan x} - \dfrac{1}{\cos x} = \dfrac{1}{\cos x} - \dfrac{1}{\sec x + \tan x}$

28. $\dfrac{\tan \theta}{1 - \cot \theta} + \dfrac{\cot \theta}{1 - \tan \theta} = 1 + \tan \theta + \cot \theta = \sec \theta \operatorname{cosec} \theta + 1$

29. $(\sin \theta + \operatorname{cosec} \theta)^2 + (\cos \theta + \sec \theta)^2 = 7 + \tan^2 \theta + \cot^2 \theta$

30. $\sec A (1 - \sin A)(\sec A + \tan A) = 1$

31. If $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$ then show that $\cos \theta - \sin \theta = \sqrt{2} \sin \theta$

32. If $\tan \theta + \sin \theta = m$, $\tan \theta - \sin \theta = n$ then show that $m^2 - n^2 = 4\sqrt{mn}$.

33. If $\sec \theta = x + \dfrac{1}{4x}$, prove that $\sec \theta + \tan \theta = 2x$ or $\dfrac{1}{2x}$

34. If $\sin \theta + \sin^2 \theta = 1$, prove that $\cos^2 \theta + \cos^4 \theta = 1$

35. Without using trigonometric table, the value of

$\cot \theta \tan(90° - \theta) - \sec(90° - \theta) \operatorname{cosec} \theta + \sin^2 65° + \sin^2 25° + \sqrt{3} \tan 5° \tan 85°$.

36. Prove that : $\dfrac{\cot(90° - \theta)}{\tan \theta} + \dfrac{\operatorname{cosec}(90° - \theta) \sin \theta}{\tan(90° - \theta)} = \sec^2 \theta$

37. Find the value of :

$$\frac{\cos 20° + \cos^2 70°}{\sec^2 50° - \cot^2 40} + 2\,\text{Cosec}^2\,58° - 2\,\text{Cot}\,58°\,\tan 32° - 4\tan 13°\,\tan 37°\,\tan 77°$$

$\tan 45°\,\tan 53°.$

38. If A, B, C are the angles of $\triangle ABC$ then prove that $\text{cosec}^2\left(\dfrac{B+C}{2}\right) - \tan^2\dfrac{A}{2} = 1$

39. Find the value of $\sec^2 10° - \cot^2 80° + \dfrac{\sin 15°\cos 75° + \cos 15°\sin 75°}{\cos\theta\sin(90° - \theta) + \sin\theta\cos(90° - \theta)}.$

40. Prove that : $\dfrac{\tan\theta - \cot\theta}{\sin\theta\cos\theta} = \tan^2\theta - \cot^2\theta.$

LONG ANSWER TYPE QUESTIONS

Prove That:

41. $\dfrac{\text{Sec}\,\theta + \tan\theta - 1}{\tan\theta - \text{Sec}\,\theta + 1} = \dfrac{\text{Cos}\,\theta}{1 - \text{Sin}\,\theta}$

42. $\left(1 + \dfrac{1}{\tan^2\theta}\right)\left(1 + \dfrac{1}{\text{Cot}^2\theta}\right) = \dfrac{1}{\text{Sin}^2\theta - \text{Sin}^4\theta}$

43. $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$

44. $(1 + \cot A + \tan A)(\sin A - \cos A) = \sin A\tan A - \cot A\cos A$

45. If $\text{Sin}\,\theta + \text{Cos}\,\theta = m$ and $\text{Sec}\,\theta + \text{Cosec}\,\theta = n$ then show that $n(m^2 - 1) = 2m$

46. find the value of :

$$\frac{\text{Cot}(90° - \theta)\tan\theta - \text{Cosec}(90° - \theta)\text{Sec}\,\theta}{\text{Sin}\,12°\,\text{Cos}\,15°\,\text{Sec}\,78°\,\text{Cosec}\,75°} + \frac{\text{Cos}^2(50° + \theta)\tan^2(40° - \theta)}{\tan 15°\,\tan 37°\,\tan 53°\,\tan 75°}$$

47. Prove that :

$$\frac{1}{\text{Cosec}\,\theta + \text{Cot}\,\theta} - \frac{1}{\text{Sin}\,\theta} = \frac{1}{\text{Sin}\,\theta} - \frac{1}{\text{Cosec}\,\theta - \text{Cot}\,\theta}$$

48. If $\dfrac{\text{Cos } \alpha}{\text{Cos } \beta} = m$ and $\dfrac{\text{Cos } \alpha}{\text{Sin } \beta} = n$, then prove that $(m^2 + n^2)\,\text{Cos}^2\,\beta = n^2$

49. If $\tan \theta + \text{Sin } \theta = m$. $\tan \theta - \sin \theta = n$, then prove that $m^2 - n^2 = 4\sqrt{mn}$

50. **Prove that :**

$$\text{Sec}^2\,\theta - \dfrac{\text{Sin}^2\,\theta - 2\text{Sin}^4\,\theta}{2\text{Cos}^4\,\theta - \text{Cos}^2\,\theta} = 1$$

51. $\text{Cot } \theta \tan(90° - \theta) - \text{Sec}(90° - \theta)\,\text{Cosec } \theta + \sqrt{3}\,\tan 12°\tan 60°\tan 78°$ find its value.

52. Find the value of —

$$\dfrac{\text{Sec}(90° - \theta)\,\text{Cosec } \theta - \tan(90° - \theta)\,\text{Cot } \theta + \text{Cos}^2\,25° + \text{Cos}^2\,65°}{3\tan 27° \tan 63°}$$

ANSWERS

1. 45°
2. 30°
3. 24°
4. $\dfrac{625}{168}$
5. 7
6. $\dfrac{1}{3}$
7. a
8. 0
9. $\dfrac{1}{5}$
10. 9
11. $\dfrac{\sqrt{1+\text{Cos}^2\,\theta}}{\text{Cot}\,\theta}$
12. 0°
13. 50°
14. $\tan^2\theta$
15. $\tan\theta$
20. A = 45°, B = 15°
21. 1
23. 22°
24. $\dfrac{17}{8}$
25. 20°
35. $\sqrt{3}$
37. -1
39. 2
46. 0
51. 2
52. $\dfrac{2}{3}$

Practice-Test

Introduction to Trigonometry

Time : 50 Minutes   *M.M.: 20*

SECTION-A

1. If $\sin \theta = \dfrac{4}{5}$ what is the value of $\cos \theta$.   1

2. Write the value of $\sin(45° + \theta) - \cos(45° - \theta)$.   1

SECTION-B

3. If $5\tan\theta = 4$ then find the value of $\dfrac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$ 2

4. Find the value of $\tan 35° \tan 40° \tan 45° \tan 50° \tan 55°$ 2

SECTION-C

5. Prove that $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = 2\,\text{Cosec}\,\theta$ 3

6. Prove that $\dfrac{\cos A}{1-\tan A} - \dfrac{\sin^2 A}{\cos A - \sin A} = \sin A + \cos A$ 3

SECTION-D

7. If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \dfrac{1}{\sqrt{3}}$ then find the value of A & B.   4

8. Prove that $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \dfrac{\cos\theta}{1-\sin\theta}$.   4

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