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Chapter

Quadratic Equations

Key Points

1. **Quadratic Equation**:- An equation of the form $ax^2 + bx + c = 0$, $a \neq 0$ is called a quadratic equation in one variable x , where a , b and c are constants.

For example $2x^2 - 3x + 1 = 0$

2. **Roots of a Quadratic Equation**:-

Let $ax^2 + bx + c = 0$, be a quadratic equation. If α is a root of this equation. It means $x = \alpha$ satisfies this equation *i.e.*, $a\alpha^2 + b\alpha + c = 0$

3. **Number of Roots**:- A quadratic equation has two roots.

4. Methods For Solving Quadratic Equation

(a) By factorization

(b) By completing the square

(c) By Quadratic Formula

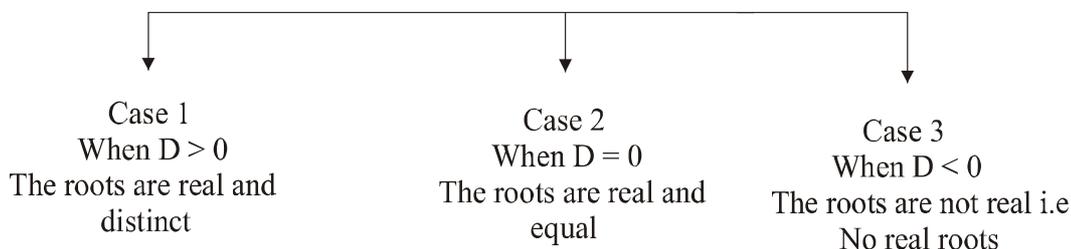
5. **Quadratic Formula** to find roots of $ax^2 + bx + c = 0$ is given by :

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a},$$

6. **Discriminant**:- For the quadratic equation $ax^2 + bx + c = 0$ the expression $b^2 - 4ac$ is called the discriminant and is denoted by D . Then the roots of the quadratic equation are given by $x = \frac{-b \pm \sqrt{D}}{2a}$

7. **Nature of Roots**

Nature of Roots



VERY SHORT ANSWER TYPE QUESTIONS

1. If $\frac{-1}{2}$ is one root of quadratic equation $2x^2 + kx + 1 = 0$, find k .
2. Find the nature of the roots of $3x^2 - 4\sqrt{3}x + 4 = 0$.
3. Is $x^3 - 4x^2 - x + 1 = (x - 2)^3$ a quadratic equation?
4. Which constant should be added and subtracted to solve the quadratic equation $5x^2 - \sqrt{2}x + 3 = 0$ by the method of completing the square?
5. If $px^2 + 3x + q = 0$ has two roots $x = -1$ $x = -2$ and find $q - p$.
6. If two roots of a quadratic equation are $\sqrt{2}$ and 1 then form the quadratic equation.
7. Represent the following in the form of a quadratic equation:- "The product of two consecutive even integers is 1848".
8. Is 0.2 a root of $x^2 - 0.4 = 0$?
9. If the quadratic equation $ax^2 + bx + c = 0$ has equal roots then find c in terms of a and b .
10. If the equation $x^2 + 6x - 91 = 0$ can be written as $(x + p)(x + q) = 0$ then find p and q .

SHORT ANSWER TYPE(I) QUESTIONS

11. Solve by factorisation method:
 - (a) $8x^2 - 22x - 21 = 0$
 - (b) $3\sqrt{5}x^2 + 25x + 10\sqrt{5} = 0$
 - (c) $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$
 - (d) $2x^2 + ax - a^2 = 0$
12. If roots of quadratic equation $2x^2 - kx + k = 0$ are real and equal, then find k .
13. Find k for which the given quadratic equation $9x^2 + 3kx + 4 = 0$ has distinct roots.
14. Find p for which the equation $x^2 + 5px + 16 = 0$ has no real roots.
15. For what value of c , roots of quadratic equation $4x^2 - 2x + (c - 4) = 0$ are reciprocal of each other.

16. For what value of p equation $px^2 + 6x + 4p = 0$ has product of root equal to the sum of roots.
17. Two squares have sides x cm and $(x + 4)$ cm. The sum of their areas is 656 cm². Find the sides of the square.
18. Find p for which the quadratic equation $px(x - 3) + 9 = 0$ have real and equal roots.
19. Divide 16 into two parts such that twice the square of the larger part exceeds the square of the smaller part by 164.
20. For what value of k , $x^2 - 5x + 3(k - 1) = 0$ has difference of roots equal to 11.
21. The sum of squares of two consecutive natural numbers is 313, find the numbers.

SHORT ANSWER TYPE (II) QUESTIONS

22. Solve the following quadratic equation:

- (a) $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$, $a + b \neq 0$
- (b) $\frac{1}{2a+b+x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$, $2a + b \neq 0$
- (c) $\frac{2}{x+1} + \frac{3}{2(x-2)} = \frac{23}{5x}$, $x \neq -1, 2, 0$
- (d) $3\left(\frac{7x+1}{5x-3}\right) - 4\left(\frac{5x-3}{7x+1}\right) = 11$ $x \neq \frac{3}{5}, \frac{-1}{7}$
- (e) $\frac{x-1}{x+2} + \frac{x-3}{x-4} = \frac{10}{3}$, $x \neq -2, 4$
- (f) $ax^2 + (4a^2 - 3b)x - 12ab = 0$
- (g) $4x^2 - 4ax + (a^2 - b^2) = 0$
- (h) $\frac{4}{x} - 3 = \frac{5}{2x+3}$, $x \neq 0, \frac{-3}{2}$

23. Using quadratic formula, solve the following.
 $abx^2 + (b^2 - ac)x - bc = 0$
24. If -5 is a root of $2x^2 + px - 15 = 0$ and roots of $p(x^2 + x) + k = 0$ are equal, then find p and k .

LONG ANSWER TYPE QUESTIONS

25. Find p for which $(p + 1)x^2 - 6(p + 1)x + 3(p + q) = 0$, $q \neq -1$, has equal roots. Hence find the roots of the equation.
26. Find k for which the quadratic equation $(2k + 1)x^2 - (7k + 2)x + (7k - 3) = 0$ has equal roots. Also find the roots.
27. If the equation $(1 + m^2)x^2 + 2mcx + (c^2 - a^2) = 0$ has equal roots, then prove $c^2 = a^2(1 + m^2)$.
28. For what value of k , $(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$ is a perfect square.
29. Out of a group of swans, $\frac{7}{2}$ times the square root of the number are playing on the sea shore of a tank. The two remaining ones are playing in the water. What is the total number of swans?
30. A peacock is sitting on the top of a pillar, which is 9 m high. From a point 27 m away from the bottom of the pillar, a snake is coming to its hole at the base of the pillar. Seeing the snake, the peacock pounces on it. If their speeds are equal, at what distance from the hole is the snake caught?
31. Rs 9000 were divided equally among a certain number of persons. Had there been 20 more persons, each would have got Rs 160 less. Find the original number of persons.
32. A dealer sells a toy for Rs 24 and gains as much percent as the cost price of the toy. Find the cost price of the toy.
33. A shopkeeper buys a number of books for Rs 80. If he had bought 4 more books for the same amount, each book would cost Rs 1 less. How many books did he buy?
34. Two pipes running together can fill a cistern in $3\frac{1}{13}$ minutes. If one pipe takes 3 minutes more than the other to fill it, find the time in which each pipe would fill the cistern?

35. A chess board contains 64 equal squares and the area of each square is 6.25 cm^2 . A border round the board is 2 cm wide. Find the length of the side of the chess board.
36. Sum of the areas of two squares is 400 cm^2 . If the difference of their perimeters is 16 cm, find the sides of two squares.
37. The area of an isosceles triangle is 60 cm^2 and the length of each one of its equal sides is 13 cm. Find its base.
38. A girl is twice as old as her sister. Four years hence the product of their ages (in years) will be 160. Find their present age.
39. A motor boat whose speed in still water is 18 km/hr takes 1 hour more to go 24 km upstream than to return down stream to the same spot. Find the speed of the stream.
40. A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/hr less than that of the fast train, find the speeds of the two trains.
41. The numerator of a fraction is 3 less than the denominator. If 2 is added to both the numerator and the denominator, then the sum of the new fraction and the original fraction is $\frac{29}{20}$. Find the original fraction.
42. The difference of two natural numbers is 3 and the difference of their reciprocals is $\frac{3}{28}$. Find the numbers.
43. Three consecutive positive integers are such that the sum of the square of the first and the product of other two is 46, find the integers.
44. A two digit number is four times the sum and three times the product of its digits. Find the numbers.
45. The hypotenuse of a grassy land in the shape of a right triangle is 1 metre more than twice the shortest side, If the third side is 7 metres more than the shortest side, find the sides of the grassy land.
46. In a class test, the sum of the marks obtained by P in Mathematics and Science is 28. Had he got 3 marks more in Mathematics and 4 marks less in Science, the product of his marks, would have been 180. Find the marks in the two subjects.

47. A piece of cloth costs Rs 200. If the piece was 5m longer and each metre of cloth costs Rs 2 less, the cost of the piece would have remained unchanged. How long is the piece and what is the original rate per metre?
48. A plane left 30 minutes later than the schedule time and in order to reach the destination 1500 km away in time it has to increase its speed by 250 km/hr from its usual speed. Find its usual speed.
49. If the sum of first n even natural numbers is 420. Find the value of n .
50. While boarding an aeroplane a passenger got hurt. The pilot showing promptness and concern, made arrangements to hospitalise the injured and so the plane started late by 30 minutes to reach the destination, 1500 km away in time, the pilot increased the speed by 100 km/hr. Find the original speed of the plane.
51. A takes 10 days less than the time taken by B to finish a piece of work. If both A and B together can finish the work in 12 days, find the time taken by B to finish the work alone.

ANSWERS

1. $k = 3$
2. The roots real & equal
3. Yes
4. $\frac{1}{50}$ or $\frac{2}{100}$
5. 1
6. $x^2 - (\sqrt{2} + 1)x + \sqrt{2} = 0$
7. $x^2 + 2x - 1848 = 0$
8. No
9. $c = \frac{b^2}{4a}$
10. 13, -7
11. (a) $x = \frac{7}{2}, x = \frac{-3}{4}$
- (b) $x = -\sqrt{5}, x = \frac{-2\sqrt{5}}{3}$
- (c) $x = \sqrt{6}, x = \frac{-\sqrt{6}}{3}$
- (d) $x = \frac{a}{2}, x = -a$
12. $k = 0, 8$
13. $k > 4, k < -4$
14. $\frac{-8}{5} < p < \frac{8}{5}$
15. $c = 8$
16. $p = \frac{-3}{2}$
17. 16 cm, 20 cm
18. $p \neq 0, p = 4$
19. $x = 10, 6$
20. $k = -7$
21. 12, 13
22. (a) $x = -a, x = -b$
- (b) $x = -a, x = \frac{-b}{2}$
- (c) $x = 4, x = \frac{-23}{11}$
- (d) $x = 0, x = 1$
- (e) $x = \frac{1 \pm \sqrt{297}}{4}$
- (f) $x = \frac{3b}{a}, x = -4a$
- (g) $x = \frac{a+b}{2}, x = \frac{a-b}{2}$
- (h) $x = -2, x = 1$

23. $x = \frac{c}{b}, x = \frac{-b}{a}$

24. $p = 7, k = \frac{7}{4}$

25. $p = 3, x = 3, 3$

26. $k = 4, \frac{-4}{7}$

28. $k = 0, 3$

29. 16

30. 12 m

31. 25

32. Rs. 20

33. 16

34. 5 minutes, 8 minutes

35. 12 cm, 16 cm

36. 24 cm or 10 cm

37 Length = 24 cm

38. 6 years, 12 years

39. 6 km/hr

40. 40 km/hr, 50 km/hr

41. $\frac{7}{10}$

42. 7, 4

43. 4, 5, 6

44. 24

45. 8 m, 17 m, 15 m

46. Marks in Maths = 12,

47. length = 20 m

Marks in science = 16

rate = Rs. 10/meter

48. 750 km/hr

49. $x = 20$

50. 500 km/hr

51. 30 days

Practice Test

Quadratic Equations

Time: 50 Minutes

M.M : 20

SECTION-A

1. If the discriminant of the quadratic equation $6x^2 - bx + 2 = 0$ is 1 then find b . 1
2. Solve $x^2 + 5x - 300 = 0$. 1

SECTION-B

3. If $kx^2 - 2kx + 6 = 0$ has equal root, find k . 2
4. Find the value of p if the roots of $x^2 + px + 12 = 0$ are in the ratio 1:3. 2

SECTION-C

5. Solve the quadratic equation
 $(x - 1)^2 - 5(x - 1) - 6 = 0$ 3
6. Find the value of k , so that the difference of roots of
 $x^2 - 5x + 3(k - 1) = 0$ is 11 3

SECTION-D

7. If the roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal then prove
 $2b = a + c$. 4
8. The sum of the squares of two natural numbers is 52. If the first number is 8 less than twice the second number, find the numbers. 4

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