# 2

*Chapter*

# Polynomials

## Key Points

1.  **Polynomial :** If $x$ is a variable, $n$ is a natural number and $a_0, a_1, a_2, a_3, \ldots\ldots\ldots a_n$ are real numbers, then $p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots + a_1 x + a_0, (a_n \neq 0)$ is called a polynomial in $x$.

2.  Polynomials of degree 1, 2 and 3 are called linear, quadratic and cubic polynomials respectively.

3.  A quadratic polynomial is an algebraic expression of the form $ax^2 + bx + c$, where $a$, $b$, $c$ are real numbers with $a \neq 0$.

4.  Zeroes of a polynomial $p(x)$ are precisely the $x$ – coordinates of the points where the graph of $y = p(x)$ intersects the $x$–axis, *i.e.,* $x = a$ is a zero of polynomial $p(x)$ if $p(a) = 0$

5.  A polynomial can have at most the same number of zeroes as the degree of the polynomial.

6.  (*i*) If one zero of a quadratic polynomial $p(x)$ is negative of the other, then coefficient of $x$ is 0.

    (*ii*) If zeroes of a quadratic polynomial $p(x)$ are reciprocal of each other, then coefficient of $x^2$ = constant term.

7.  Relationship between zeroes and coefficients of a polynomial

    If $\alpha$ And $\beta$ Are zeroes of $p(x)$ $ax^2 + bx + c$ ($a \neq 0$), then

    Sum of zeroes $= \alpha + \beta = -\dfrac{b}{a}$

    Product of zeroes $= \alpha\beta = \dfrac{c}{a}$

8.  If $\alpha$, $\beta$ are zeroes of a quadratic polynomial $p(x)$, then

    $p(x) = k [x^2 - (\text{sum of zeroes}) x + \text{product of zeroes}]$

    $\Rightarrow p(x) = k [x^2 - (\alpha + \beta)x + \alpha\beta]$; where $k$ is any non-zero real number.

9.  Graph of linear polynomial $p(x) = ax + b$ is a straight line.

10. Division Algorithm states that given any polynomials $p(x)$ and $g(x)$, there exist polynomial $q(x)$ and $r(x)$ such that:

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$p(x) = g(x) . q(x) + r(x)$ ; $g(x) \neq 0$,

[where either $r(x) = 0$ or degree $r(x) <$ degree $g(x)$]

## VERY SHORT ANSWER TYPE QUESTIONS

1. What will be the number of zeroes of a linear polynomial $p(x)$ if its graph (*i*) passes through the origin. (*ii*) doesn't intersect or touch $x$-axis at any point?

2. Find the quadratic polynomial whose zeroes are
$\left(5 + 2\sqrt{3}\right)$ and $\left(5 - 2\sqrt{3}\right)$

3. If one zero of $p(x) = 4x^2 - (8k^2 - 40k) x - 9$ is negative of the other, find values of $k$.

4. What number should be added to the polynomial $x^2 - 5x + 4$, so that 3 is a zero of polynomial so obtained.

5. How many (*i*) maximum (*ii*) minimum number of zeroes can a quadratic polynomial have?

6. What will be the number of real zeroes of the polynomial $x^2 + 1$?

7. If $\alpha$ and $\beta$ are zeroes of polynomial $6x^2 - 7x - 3$, then form a quadratic polynomial where zeroes are $2\alpha$ and $2\beta$

8. If $\alpha$ and $\dfrac{1}{\alpha}$ are zeroes of $4x^2 - 17x + k - 4$, find value of $k$.

9. What will be the number of zeroes of the polynomials whose graphs are parallel to (*i*) $y$-axis (ii) $x$-axis

10. What will be number of zeroes of the polynomials whose graphs are either touching or intersecting the axis only at the points:
(*i*) $(-3, 0), (0, 2)$ & $(3, 0)$ (*ii*) $(0, 4), (0, 0)$ and $(0, -4)$

## SHORT ANSWER TYPE (I) QUESTIONS

11. If $-3$ is one of the zeroes of the polynomial $(k - 1)x^2 + k x + 1$, find the value of $k$.

12. If the product of zeroes of $ax^2 - 6x - 6$ is 4, find the value of a. Hence find the sum of its zeroes.

13. If $\alpha$ and $\beta$ are zeroes of the polynomial $x^2 - a(x + 1) - b$ such that $(\alpha + 1)(\beta + 1) = 0$, find the value of $b$.

14. If zeroes of $x^2 - kx + 6$ are in the ratio 3 : 2, find $k$.

15. If one zero of the quadratic polynomial $(k^2 + k)x^2 + 68x + 6k$ is reciprocal of the other, find $k$.

16. If $\alpha$ and $\beta$ are the zeroes of the polynomial $x^2 - 5x + m$ such that $\alpha - \beta = 1$, find $m$.

17. If the sum of squares of zeroes of the polynomial $x^2 - 8x + k$ is 40, find the value of $k$.

18. If $\alpha$ and $\beta$ are zeroes of the polynomial $t^2 - t - 4$, form a quadratic polynomial whose zeroes are $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$.

## SHORT ANSWER TYPE (II) QUESTIONS

19. If $(k + y)$ is a factor of each of the polynomials $y^2 + 2y - 15$ and $y^3 + a$, find values of $k$ and $a$.

20. Obtain zeroes of $4\sqrt{3}\, x^2 + 5x - 2\sqrt{3}$ and verify relation between its zeroes and coefficients.

21. If $x^4 + 2x^3 + 8x^2 + 12x + 18$ is divided by $(x^2 + 5)$, remainder comes out to be $(px + q)$, find values of $q$ and $q$.

22. $-5$ is one of the zeroes of $2x^2 + px - 15$, zeroes of $p(x^2 + x) + k$ are equal to each other. Find the value of $k$.

23. Find the value of $k$ such that $3x^2 + 2kx + x - k - 5$ has the sum of zeroes as half of their product.

24. If $\alpha$ and $\beta$ are zeroes of $y^2 + 5y + m$, find the value of $m$ such that $(\alpha + \beta)^2 - \alpha\beta = 24$

25. If $\alpha$ and $\beta$ are zeroes of $x^2 - x - 2$, find a polynomial whose zeroes are $(2\alpha + 1)$ and $(2\beta + 1)$

26. Find values of $a$ and $b$ so that $x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$.

27. What must be subtracted from $8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $4x^2 + 3x - 2$ ?

28. What must be added to $4x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is divisible by $x^2 - 2x - 3$ ?

## LONG ANSWER TYPE QUESTIONS

**29.** Find all zeroes of the polynomial $2x^3 + x^2 - 6x - 3$ if two of its zeroes are $\sqrt{3}$ and $-\sqrt{3}$.

**30.** If $\sqrt{2}$ is a zero of $(6x^3 + \sqrt{2}\,x^2 - 10x - 4\sqrt{2})$, find its other zeroes.

**31.** If two zeroes of $x^4 - 6x^3 - 26x^2 + 138\,x - 35$ are $(2 \pm \sqrt{3})$, find other zeroes.

**32.** On dividing the polynomial $x^3 - 5x^2 + 6x - 4$ by a polynomial $g(x)$, quotient and remainder are $(x - 3)$ and $(-3x + 5)$ respectively. Find $g(x)$

**33.** If sum and product of two zeroes of the polynomial $x^3 + x^2 - 3x - 3$ are 0 and 3 respectively, find all zeroes of the polynomial.

**34.** If $-\dfrac{1}{2}$ is a zero of the polynomial $2x^3 + x^2 - 6x - 3$, find the sum and product of its other two zeroes.

**35.** Obtain all zeroes of the polynomial $2x^4 - 2x^3 - 7x^2 + 3x + 6$ if two factors of this polynomial are $\left( x \pm \sqrt{\dfrac{3}{2}} \right)$.

**36.** Sum and product of two zeroes of $x^4 - 4x^3 - 8x^2 + 36x - 9$ are 0 and $-9$ respectively. Find the sum and product of its other two zeroes.

**37.** A person distributes $k$ books to some needy students. If $k$ is a zero of the polynomial $x^2 - 100x - 20000$, then

  *(i)* Find the number of books distributed

**38.** One zero of $x^3 - 12x^2 + 47x - 60$ is 3 and the remaining two zeroes are the number of trees planted by two students.

  *(i)* Find the total number of trees planted by both students.

**ANSWERS**

1. (i) 1 (ii) 0
2. $x^2 - 10x + 13$
3. $k = 0, 5$
4. 2
5. (i) 2 (ii) 0
6. 0
7. $3x^2 - 7x - 6$
8. $k = 8$
9. (i) 1 (ii) 0
10. (i) 2 (ii) 1
11. $\dfrac{4}{3}$
12. $a = -\dfrac{3}{2}$, sum of zeroes $= -4$
13. 1
14. $-5, 5$
15. 5
16. 6
17. 12
18. $4t^2 + t - 1$
19. $k = 3, -5$ and $a = 27, -125$
20. $-\dfrac{2}{\sqrt{3}}, \dfrac{\sqrt{3}}{4}$
21. $p = 2, q = 3$
22. $\dfrac{7}{4}$
23. 1
24. 1
25. $x^2 - 4x - 5$
26. $a = 1$, $b = 7$
27. $14x - 10$
28. $61x - 65$
29. $\sqrt{3}, -\sqrt{3}, -\dfrac{1}{2}$
30. $-\dfrac{\sqrt{2}}{2}, -\dfrac{2\sqrt{2}}{3}$
31. $-5, 7$
32. $x^2 - 2x + 3$
33. $\sqrt{3}, -\sqrt{3}, -1$
34. 0, 3
35. $2, -1, \pm\sqrt{\dfrac{3}{2}}$
36. 4, 1
37. (i) 200
38. (i) 9

## Practice-Test

## Polynomials

*Time : 50 Minutes*                                                    *M.M. : 20*

### SECTION-A

1. If $\alpha$ and $\beta$ are zeroes of a quadratic polynomial $p(x)$, then factorize $p(x)$. **1**

2. If $\alpha$ and $\beta$ are zeroes of $x^2 - x - 1$, find the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$. **1**

### SECTION-B

3. If $\alpha$ and $\beta$ are zeroes of $x^2 - (k + 6)x + 2(2k - 1)$. find the value of $k$ if $\alpha + \beta = \dfrac{1}{2}\alpha\beta$. **2**

4. Find a quadratic polynomial one of whose zeroes is $(3 + \sqrt{2})$ and the sum of its zeroes is 6. **2**

### SECTION-C

5. Find values of $a$ and $b$ if $(x^2 + 1)$ is a factor of the polynomial $x^4 + x^3 + 8x^2 + ax + b$. **3**

6. If truth and lie are zeroes of the polynomial $px^2 + qx + r$, $(p \neq 0)$ and zeroes are reciprocal to each other, **3**
   (*i*) Find the relation between $p$ and $r$.

### SECTION-D

7. On dividing the polynomial $x^3 + 2x^2 + kx + 7$ by $(x - 3)$, remainder comes out to be 25. Find quotient and the value of k. Also find the sum and product of zeroes of the quotient so obtained. **4**

8. If $\beta$ and $\dfrac{1}{\beta}$ are zeroes of the polynomial $(\alpha^2 + \alpha)x^2 + 61x + 6\alpha$, find values of $\beta$ and $\alpha$. **4**

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